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The sensitivity of endpoint forces produced by the extrinsic muscles of the thumb to posture

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A B S T R A C T

This study utilizes a biomechanical model of the thumb to estimate the force produced at the thumb-tip by each of the four extrinsic muscles. We used the principle of virtual work to relate joint torques produced by a given muscle force to the resulting endpoint force and compared the results to two separate cadaveric studies. When we calculated thumb-tip forces using the muscle forces and thumb postures described in the experimental studies, we observed large errors. When relatively small deviations from experimentally reported thumb joint angles were allowed, errors in force direction decreased substantially. For example, when thumb posture was constrained to fall within $\pm 15^\circ$ of reported joint angles, simulated force directions fell within experimental variability in the proximal–palmar plane for all four muscles. Increasing the solution space from $\pm 1^\circ$ to an unbounded space produced a sigmoidal decrease in error in force direction. Changes in thumb posture remained consistent with a lateral pinch posture, and were generally consistent with each muscle's function. Altering thumb posture alters both the components of the Jacobian and muscle moment arms in a nonlinear fashion, yielding a nonlinear change in thumb-tip force relative to muscle force. These results explain experimental data that suggest endpoint force is a nonlinear function of muscle force for the thumb, support the continued use of methods that implement linear transformations between muscle force and thumb-tip force for a specific posture, and suggest the feasibility of accurate prediction of lateral pinch force in situations where joint angles can be measured accurately.

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1. Introduction

The opposable thumb has helped humans develop fine motor skills, allowing manipulation of a wide variety of objects. In particular, the ability to hold an object between the thumb and the lateral aspect of the index finger (lateral pinch) is an important component of hand function. For example, the ability to complete selected functional tasks has been accurately predicted for individuals with significant deficits in lateral pinch strength based on the magnitude of pinch force they could produce (Smaby et al., 2004).

The thumb consists of the carpometacarpal (CMC), metacarpophalangeal (MCP), and interphalangeal (IP) joints, connecting four bones, the trapezium, the first metacarpal, the proximal phalanx, and the distal phalanx. Anatomical studies describe five degrees of freedom among these three joints, including flexion/extension and abduction/adduction at the CMC joint, flexion/extension and abduction/adduction at the MCP joint, and flexion/extension at the IP joint (Hollister et al., 1992; Giurintano et al., 1995; Hollister et al., 1995). Each degree of freedom has a single axis of rotation, and these axes have been demonstrated to be non-orthogonal and non-intersecting. Nine muscles actuate the thumb; the five intrinsic muscles of the thumb both originate and insert within the hand and the four extrinsic muscles originate in the forearm.

Despite the existence of biomechanical models of the thumb that replicate experimental measurements of muscle moment arms (Valero-Cuevas et al., 2003; Holzbaur et al., 2005; Towles et al., 2008; Vigouroux et al., 2009; Wu et al., 2009), thumb models do not accurately predict lateral pinch force (Valero-Cuevas et al., 2003). Notably, the mathematical transformation between muscle force and endpoint force used in biomechanical models is linear for a given posture (Valero-Cuevas et al., 2000). In contrast, experimental measurements obtained from cadaveric specimens suggest that endpoint force is a nonlinear function of muscle force for the thumb (Pearlman et al., 2004). That is, when different levels of muscle force were applied while the thumb was in the same initial posture, nonlinear changes in endpoint force

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were observed. Pearlman et al. attributed the experimentally observed nonlinearities to load-dependent viscoelastic tendon paths and load-dependent motion of the trapezium. In a separate study, Towles et al. (2008) illustrated that, theoretically, translation of the trapezium can cause the thumb to assume a different overall posture.

The main objective of this study is to evaluate the sensitivity of endpoint forces produced by the four extrinsic muscles of the thumb to posture. The extrinsic muscles are flexor pollicis longus (FPL), extensor pollicis longus (EPL), extensor pollicis brevis (EPB), and abductor pollicis longus (APL). We hypothesized that the errors between endpoint forces produced by individual muscles simulated using a biomechanical model (Holzbaur et al., 2005) and measured experimentally in two studies (Pearlman et al., 2004; Towles et al., 2004) would decrease if deviations from the experimentally reported joint angles were allowed. In both experimental studies, joint angle measurements were performed manually before the muscles were loaded and it was noted anecdotally that thumb posture visibly changed during data collection. Because the thumb always maintained a posture that was consistent with lateral pinch in these experimental studies, we hypothesized that the deviations in overall posture required to produce acceptable errors between simulated and measured endpoint forces would be relatively small.

2. Methods

To estimate the force produced at the thumb-tip by the individual extrinsic muscles, we used the principle of virtual work to relate the joint torques produced by a given muscle force to the resulting force at a point of interest on the distal phalanx:

$$\overline{F} = (\boldsymbol{J}\boldsymbol{J}^T)^{-1}\boldsymbol{J}\overline{L}_{MA}f_{muscle}, \quad (1)$$

where $\overline{F}$ is the endpoint force, $\boldsymbol{J}$ the 3×4 Jacobian matrix, $\overline{L}_{MA}$ the 4×1 vector of muscle moment arms, and f_{muscle} the given muscle force of the extrinsic muscle of interest. In this study, we specified the magnitude of a given muscle force, f_{muscle}, to replicate the load applied to that muscle in the experimental studies described above.

The remaining components of Eq. (1) were developed using a kinematic model of the thumb, with the goal of transforming muscle force into a pure point force at the thumb-tip, assuming any moments caused by the distal phalanx slipping or rotating to be equal to zero. The thumb model has been described previously as a part of a general musculoskeletal model of the upper limb, including bone geometry, joint kinematics, and muscle–tendon paths consistent with a healthy, 50th percentile male (Holzbaur et al., 2005). The thumb is modeled as four separate hinge joints, with the axes and centers of rotation for the thumb joints based on detailed experimental studies (Fig. 1) (Hollister et al., 1992, 1995). Specifically, four independent angular rotations occur about non-intersecting, non-orthogonal axes of rotation where all abduction and adduction thumb movement occurs at the CMC joint.

To convert the force specified for a given muscle into the thumb joint torques it produces, we used the musculoskeletal model to calculate the moment arm of each of the extrinsic muscles about each axis of rotation over the full range of motion. The muscle moment arms estimated using the biomechanical model are consistent with experimental data (Smutz et al., 1998). The moment arms for a given muscle defined the 4×1 vector, $\overline{L}_{MA}$, which varies as a function of joint posture.

To derive the Jacobian, $\boldsymbol{J}$, we first calculated the three-dimensional position of the thumb-tip with respect to the wrist center as a function of the four degrees of freedom of the thumb. We then calculated the partial derivative of each position component with respect to each degree of freedom. The resulting 3×4 matrix is dependent on the axes of rotation (defined above), segment lengths, and joint angles from the thumb model. The segment lengths were determined from the musculoskeletal model using the distances between the centers of rotation for the thumb joints. Joint angles were measured relative to the neutral position of the thumb, as defined in Holzbaur et al. (2005). The Moore–Penrose pseudoinverse was used to calculate $\boldsymbol{J}^{-T}$ due to the non-square nature of the matrix.

Towles et al. (2004) reported thumb-tip force vectors produced when the FPL was loaded with 10 N of force in 7 cadaveric specimens. We first performed 7 simulations to estimate the transformation from FPL muscle force to thumb-tip force for each specimen. In these simulations, we used the joint angles reported in the experimental study, which differed for each specimen. We then calculated the 7 thumb postures that resulted in the smallest absolute errors compared with the

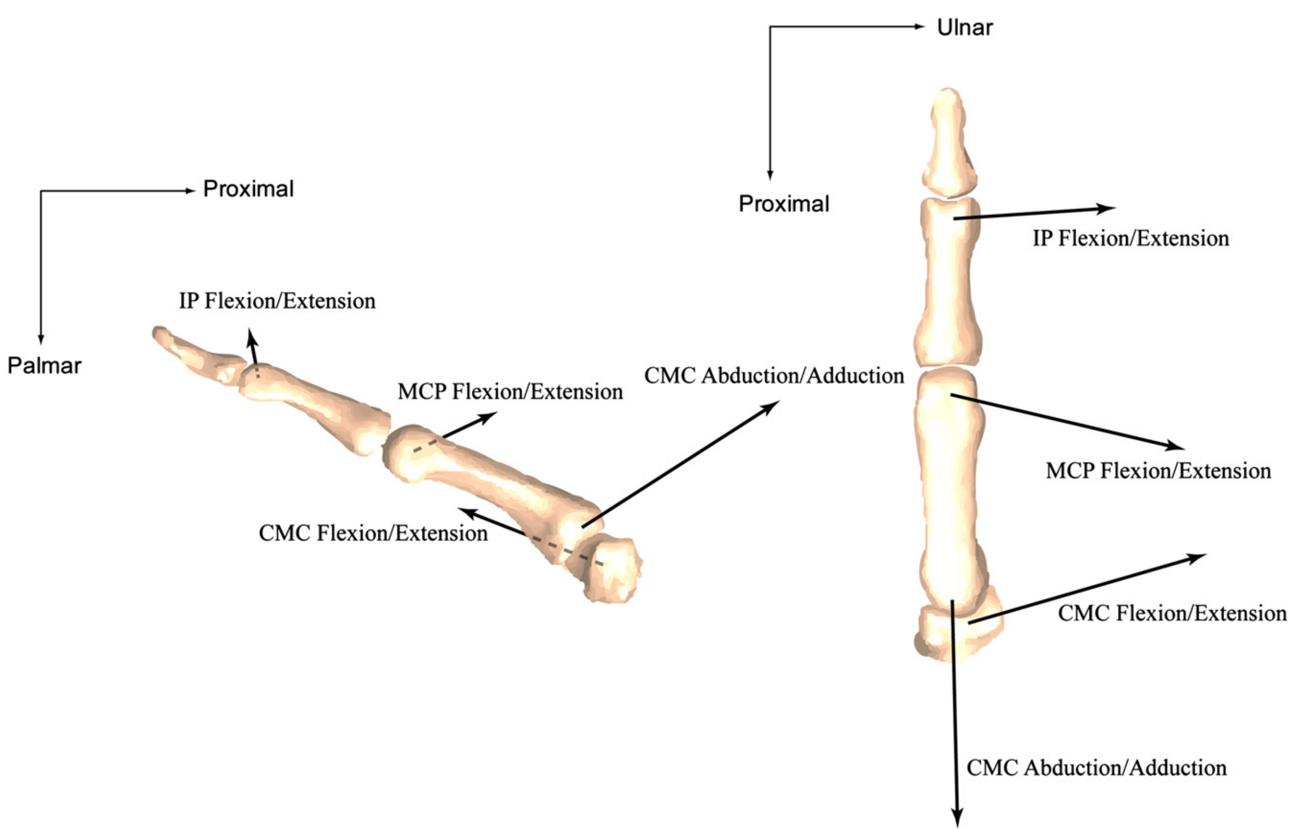

Fig. 1. The axes and centers of rotation for the thumb joints used in the musculoskeletal computer model. Note that all abduction/adduction movement of the thumb model occurs at the CMC joint.

7 measured force vectors. To do so, we positioned the thumb in neutral abduction/adduction and examined the entire set of possible thumb postures by varying the positions of the CMC, MP, and IP joints in 1° increments across the full range of flexion/extension. For each specimen, we then calculated the errors between the measured force vector and force vectors simulated at every possible configuration of the joints of the thumb. From these calculations, we identified the set of thumb postures that produced an error in force direction in the proximal–palmar plane that was less than 5°. From that set of postures, we identified the thumb postures that fell within a specified range (±20° flexion/extension at each joint) of the reported thumb posture. This subset of postures was further examined to identify the thumb posture that had the minimum error in force magnitude, evaluated to 0.01 N. This thumb posture was then defined as the "optimal" thumb posture. We performed a matched pairs *t*-test to determine if the errors in the simulation results for these postures were less than the errors for the experimental joint postures across the 7 specimens. Results were considered significant for $p<0.05$.

Summarizing data collected from 13 cadaveric specimens, Pearlman et al. (2004) reported mean thumb-tip force vectors when individual thumb muscles were loaded to 30% of their maximum isometric force-generating capacity: 26.1 N of force was applied to the FPL, 12.6 N to the EPL, 7.8 N to the EPB, and 30 N to the APL. We performed one simulation for each extrinsic muscle using the experimentally reported joint angles; in the Pearlman study, a single set of joint angles representing the average joint posture of the 13 specimens was reported. We then calculated the 4 thumb postures that resulted in the smallest absolute errors compared with the 4 measured force vectors for the FPL, EPL, EPB, and APL, using a method similar to what is outlined above. For three of the four muscles studied, the set of thumb postures that produced an error in force direction less than 5° was bounded by ±30° flexion/extension at each joint. For one muscle (EPL), none of the thumb postures examined produced an error in force direction less than 5°. As a result, we identified the posture with the smallest possible error. While our method concentrates on minimizing the error in force direction in the proximal–palmar plane, the error in force direction between our simulations and the data reported by Pearlman et al. were evaluated in both the proximal–palmar and the proximal–ulnar planes. Because the results for individual cadaveric specimens were not reported by Pearlman et al. (2004), we evaluated the simulation results relative to the experimental variability reported for force direction (34°) and magnitude (35% of the mean).

The effects of small changes in joint angles (≤20°) on the endpoint force in the proximal–palmar plane were examined by repeating the simulations for the four extrinsic muscles within a constrained set of thumb postures. Five sets of bounded simulations constrained the range of joint postures we explored to ±1°, ±5°, ±10°, ±15°, and ±20° from the reported joint angles at each flexion/extension joint. We then searched through every combination of flexion/extension joint angle for each of these sets of bounded simulations and determined the set of joint angles that first minimized the error in force direction and then minimized the error in force magnitude, similar to the method described above. This set of joint angles was defined as the optimal thumb posture for the given bounded simulation.

3. Results

When we calculated the thumb-tip force produced by the FPL using the muscle force and joint angles reported by Towles et al., we observed large errors in force magnitude and direction (Table 1). Using the optimal posture (as defined in Section 2) instead of the reported posture for each specimen decreased the error in force direction by an average of 46° (p=0.0006). On average, the directions of the force vectors calculated using the seven specimen-specific optimal postures were within 1° of the force directions reported by Towles et al. (Table 1, Fig. 2). Qualitatively, the optimal thumb postures that we identified remained consistent with a lateral pinch posture, and were comparable to the joint postures described in the cadaveric study (Fig. 3). The average (SD) differences between the optimal and

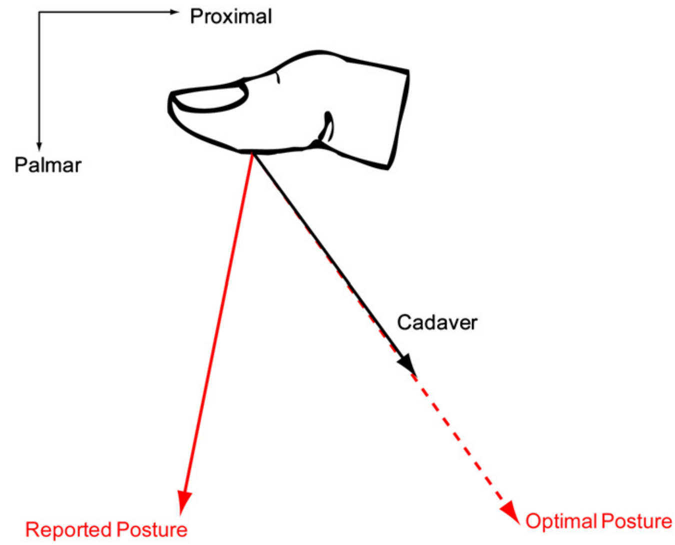

Fig. 2. Thumb-tip force produced in the proximal–palmar plane when a load of 10 N was applied to the tendon of the FPL. The black arrow indicates the average force measured from 7 cadaveric specimens by Towles et al. (2004). The solid red arrow indicates the average simulation results when the reported joint angles were used in the Jacobian. The dashed red arrow indicates the average simulation results using the optimal joint angles (for interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

reported postures were 11.14 (5.90)° for the CMC joint, 18.14 (0.38)° for the MP joint, and 16.71 (5.62)° for the IP joint. Unlike the error in force direction, however, we did not observe significant changes in the error in force magnitude with posture ($p<0.16$). The average force magnitude of the seven simulation results for FPL was over 80% larger than the mean of the results reported by Towles et al., regardless of which joint angles were used (Table 1).

When we calculated the thumb-tip forces produced by the four extrinsic muscles of the thumb using the muscle forces and thumb posture reported by Pearlman et al., we observed errors in the orientation of the force vectors in both the proximal–palmar and the ulnar-proximal planes (Table 2). When we implemented the optimal posture (identified for each muscle individually by minimizing the error observed in the proximal–palmar plane only), the directions of the thumb-tip forces fell within the reported experimental error *in both planes*, with the exception of EPB. Using the thumb posture reported by Pearlman et al., the force direction for the APL exhibited the largest initial error in both planes; using the optimal posture for APL reduced the error in both planes to within the reported experimental variability (Fig. 4). The optimal posture for EPL resulted in the largest final error in force direction in the proximal–palmar plane (Table 2), but also fell within the reported experimental variability. While the optimal posture for the EPB resulted in an error in force direction that fell outside the reported experimental variability in the ulnar-proximal plane, the error in this plane was reduced by 20° compared with the simulation that implemented the reported joint posture. In addition, the force direction for EPB in the proximal–palmar plane was within 1° of the reported direction using the optimal joint posture. For each muscle studied, the absolute difference between the reported posture and the optimal posture was less than 20° for each of the three joints, except for the MCP joint for the APL simulation (Table 3).

As we observed in the comparisons with the Towles study, the magnitudes of the thumb-tip forces simulated in this study were generally larger than those measured by Pearlman et al. and

Table 1
Mean errors in simulated thumb-tip force for the FPL.

	Reported angles	Optimized angles
Average (SD) error in direction (deg)[a]	47.47 (18.80)	0.7707 (0.2645)
Average (SD) error in magnitude (%)[b]	82.35 (69.94)	109.75 (105.10)

[a] Mean direction error $= (\frac{1}{7}) \sum_{i=1}^{7} \left| \text{reported direction}_i\text{-simulated direction}_i \right|$.

[b] Mean magnitude error $= (\frac{1}{7}) \sum_{i=1}^{7} \left| \frac{\text{Reported magnitude}_i\text{-simulated magnitude}_i}{\text{Reported magnitude}_i} \right|$.

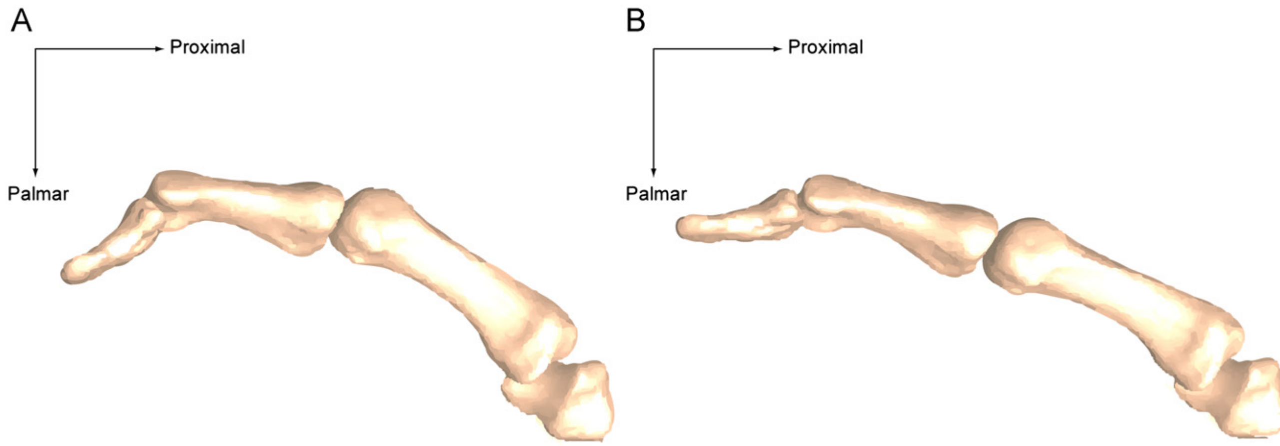

Fig. 3. (A) The average thumb posture of the seven cadaveric specimens, as reported by Towles et al. (2004). (B) The average of the seven thumb postures calculated in this study.

Table 2
Errors in simulated force magnitude and direction for the four extrinsic muscles.

	Direction[a]				Magnitude[b]	
	Proximal–palmar plane		Ulnar-proximal plane		Reported angles (%)	Optimal angles (%)
	Reported angles (deg)	Optimal angles (deg)	Reported angles (deg)	Optimal angles (deg)		
FPL	20.06	0.09	55.63	21.65	9.55	10.75
EPL	91.90	29.68	25.33	11.44	119.60	140.53
EPB	29.72	0.73	71.93	52.56	32.56	35.23
APL	139.43	1.00	97.65	10.27	14.70	48.45

[a] Direction error $= |\text{reported direction-simulated direction}|$.

[b] Magnitude error $= \left|\frac{\text{Reported magnitude-simulated magnitude}}{\text{Reported magnitude}}\right|$.

magnitude errors were not improved by changing joint posture. The force magnitudes for three of the four extrinsic muscles fell within the reported experimental error when we simulated the joint angles reported by Pearlman et al. (Table 2). However, only the FPL fell within experimental error for magnitude in the joint angles that minimized error for force direction. The optimal posture for EPL resulted in the largest final error in force magnitude (Table 2).

When the solution space for optimal thumb posture was constrained, we observed a sigmoidal decrease in the error in force direction in the proximal–palmar plane with increasing solution space for each of the four extrinsic muscles (Fig. 5). For three of the four muscles, the error was reduced to less than 50% of the initial error when thumb posture was required to fall within $\pm 10^{\circ}$ of the reported joint angles. When this constraint was broadened to $\pm 15^{\circ}$, the simulated force direction fell within the reported experimental variability in the proximal–palmar plane for all four muscles. The optimal postures identified from the $\pm 5^{\circ}$, $\pm 10^{\circ}$, and $\pm 15^{\circ}$ bounded simulations were consistent with lateral pinch for all four extrinsic muscles (Fig. 6). As the constraints were increased, the changes in thumb postures for each muscle tended toward movements consistent with the primary function of that particular muscle, for example the thumb flexed in the FPL simulations while it extended in the EPB simulations.

4. Discussion

Simulations using a three-dimensional model of the thumb produced thumb-tip forces that fell within experimental error for thumb-tip force directions when relatively small deviations from experimentally reported thumb posture were allowed. Adjustments to thumb posture in the proximal–palmar plane reduced errors in force direction in both the proximal–palmar and ulnar-proximal planes because of the non-orthogonality of the individual axes of rotation. Unlike force direction, errors in force magnitude did not improve when using the optimal joint postures.

The transformation between muscle force and thumb-tip force described by Eq. (1) involves a linear relationship. That is, given a single thumb posture and set of muscle moment arms, an increase in the muscle force results in a linear increase in the endpoint force (cf. blue and red arrows, Fig. 7A). However, Pearlman et al. observed nonlinear changes in endpoint force when different muscle forces were applied in the same initial thumb posture. We believe the experimentally observed non-linearity arose because loading the muscle changed the joint posture of the thumb slightly, and the extent of these postural changes varied with the loading condition. While Eq. (1) transforms muscle force to thumb-tip force linearly if thumb posture is constant, altering thumb posture alters both the Jacobian components and the muscle moment arms in a nonlinear fashion, changing the overall transformation from muscle force to endpoint forces. If an increase in muscle force results in a change in joint posture, a nonlinear increase in the endpoint force is observed (cf. blue and red arrows, Fig. 7B).

The errors in force direction we observed when the simulations were performed with the experimentally reported joint angles were substantially reduced if deviations in thumb posture were allowed, even when the deviations were constrained by as

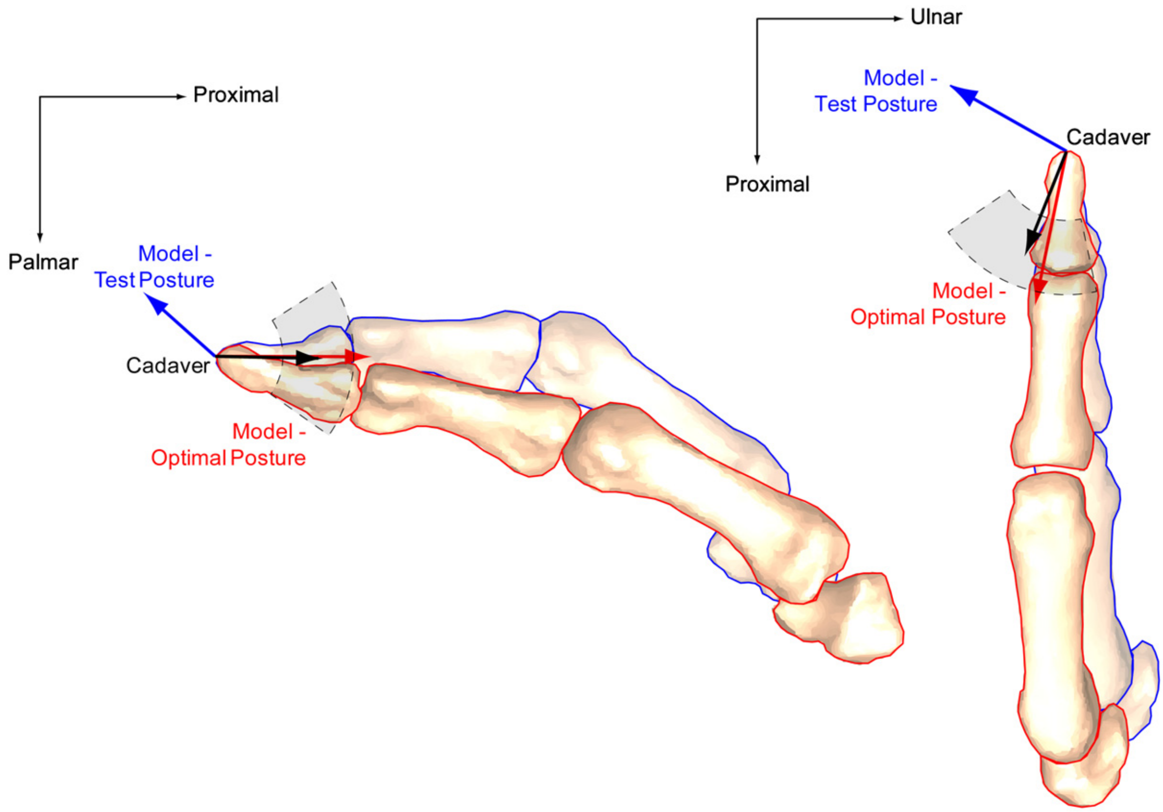

Fig. 4. Thumb-tip force produced in the proximal–palmar and ulnar-proximal planes when a load of 30 N was applied to the tendon of the APL. The black arrows indicate the average force measured from 13 cadaveric specimens by Pearlman et al. (2004). The shaded grey region represents the experimental variability region from that study. The blue arrows indicate the simulation result when the reported joint angles (thumb outlined in blue) were used in Eq. (1). The red arrows indicate the simulation result when the optimal joint angles (thumb outlined in red) were used (for interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

Table 3
Absolute errors in the flexion/extension joint angles between simulated and reported postures for the four extrinsic muscles of the thumb.

Muscles	CMC (deg)	MP (deg)	IP (deg)
FPL	14	9	14
EPL	14	0	5
EPB	15	19	3
APL	10	29	9
Average	13.25	14.25	7.75
Standard deviation	2.22	12.53	4.86

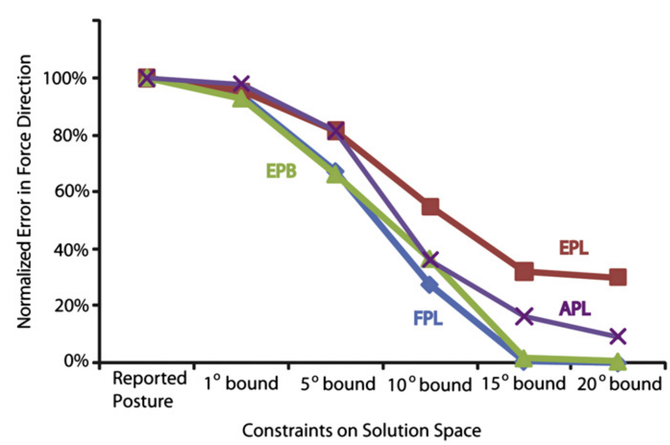

Fig. 5. The error in force direction in the proximal–palmar plane for the four extrinsic muscles of the thumb as a function of increasing solution space for the bounded simulations. The errors in force direction are normalized by the initial errors between the measure force directions and the force directions from the simulations using the experimentally reported joint postures.

little as $\pm 10^{\circ}$ at each joint (Fig. 5). When deviations of $\pm 15^{\circ}$ were allowed, simulated force directions fell within the reported experimental variability in the proximal–palmar plane for each of the four extrinsic muscles. Towles et al. (2004) reported standard deviations in joint angles as large as 10° among the seven specimens studied. Further, both experimental studies anecdotally reported that thumb posture visibly changed during the thumb-tip force measurements, indicating it is reasonable to explore changes in thumb postures that extend beyond inter-specimen variability in the posture of the unloaded thumb. Importantly, the optimal postures that resulted from both our unbounded simulations (cf. Figs. 3 and 4), and from our bounded simulations (cf. Fig. 6) remained consistent with lateral pinch. Comparing simulations under the same loading conditions and different, but similar, thumb configurations illustrates the sensitivity of thumb-tip forces to joint posture, replicates the non-linearity observed experimentally, and emphasizes the need to synchronize measurements of joint angles and thumb-tip forces in future studies.

Changes in thumb posture did not improve errors in our simulation results for force magnitude. However, the linear relationship that exists between muscle force and endpoint force in a single joint posture also provides a mechanism to alter force magnitude without affecting direction. Specifically, force magnitude can be reduced independent of direction by uniformly

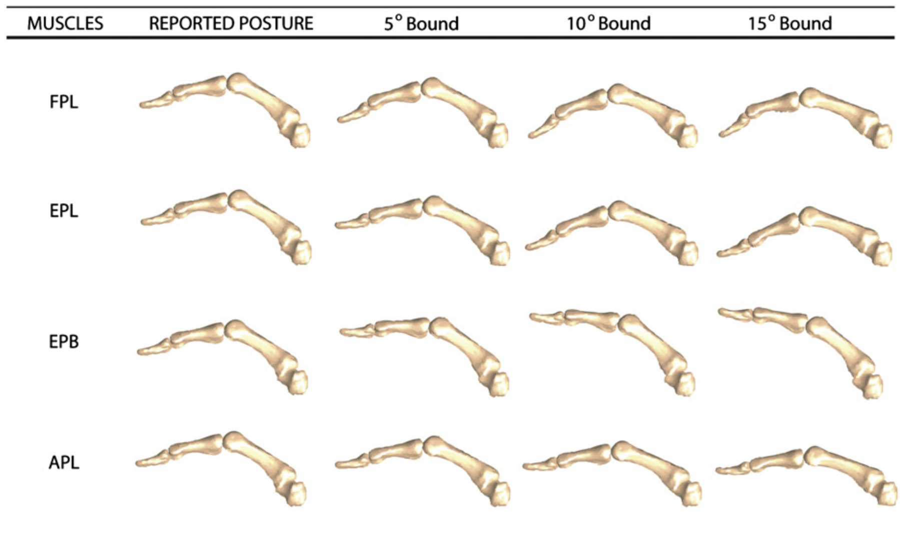

Fig. 6. Comparison of the optimal thumb postures identified for each muscle across the bounded simulations. Note that the reported posture is the same for all four muscles.

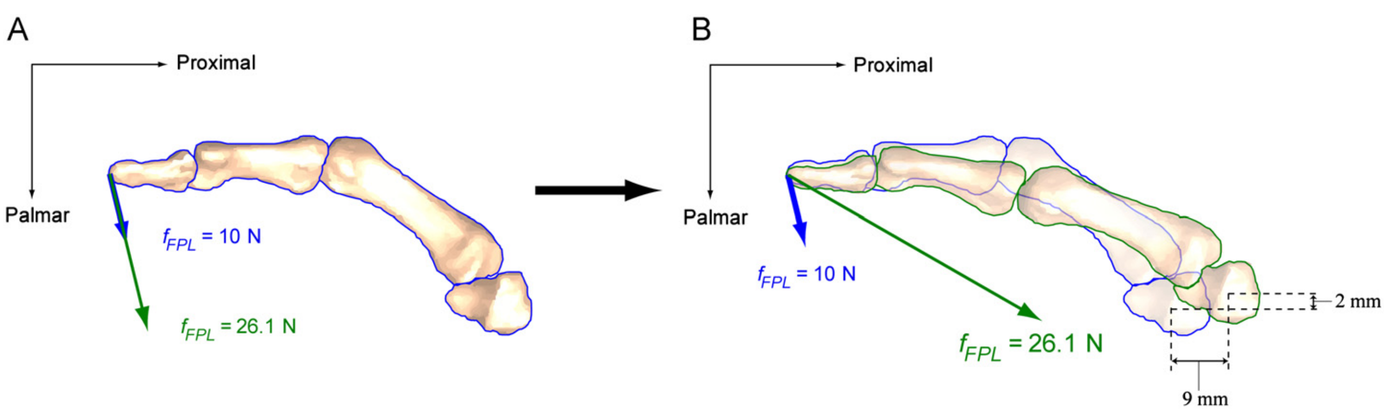

Fig. 7. (A) Illustration of the linear relationship between muscle force and thumb-tip force that is implemented with these simulation methods when joint posture remains constant under different loading conditions. (B) If a change in joint posture accompanies a change in loading conditions, the simulations do predict a nonlinear change in thumb-tip force, as has been observed experimentally.

scaling the muscle moment arms in Eq. (1). For example, uniformly decreasing muscle moment arms by 10% reduced the errors in force magnitude to within the reported experimental error for three of the four extrinsic muscles (FPL, EPB, and APL). The error for the EPL also decreased by 24%; however force magnitude still fell outside the experimental error. It is not unreasonable to assume the moment arms of the musculoskeletal model (which represents a 50th percentile male) were larger than the average for the cadaveric specimens, which included both males and females of varying sizes. Variability in EPL moment arms at the CMC joint measured in 7 cadaveric hands (5 males, 2 females) was especially large; standard deviations were on the order of 30% of the mean for flexion/extension and 75% of the mean for abduction/adduction (Smutz et al., 1998).

Our simulation study both explains experimental data that suggest endpoint force is a nonlinear function of muscle force for the thumb and supports the continued use of biomechanical modeling methods that implement linear transformations between muscle force and thumb-tip force for a specific posture. As described above, musculoskeletal parameters such as muscle moment arms do have an important impact on biomechanical simulations of thumb-tip forces. However, our work emphasizes the importance of quantifying thumb postures accurately, an experimentally challenging problem in both anatomical and human subject studies given the small size and anatomical complexity of the thumb, as well as the kinematic redundancy of the lateral pinch posture. Our simulation results provide strong support for continued development of biomechanical models of the thumb and suggest the feasibility of accurate prediction of lateral pinch force development in human subjects in situations where joint angles can be measured accurately.

Conflict of interest statement

None.

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